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Non-monotonic Magnetoresistivity in Two-dimensional Electron Systems

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The connection being studied is the one between the non-monotonic magnetoresistivity (MR) and the electron-electron interaction (EEI) correction in weakly-disordered two-dimensional electron systems (2DESs) in the ballistic region $k_BT\tau/\hbar > 1$, where k_B, T, τ, and $\hbar$ are the Boltzmann constant, the temperature, the scattering time, and the reduced Planck constant, respectively. At zero magnetic field, a transition of the resistivity $\rho(T)$ from the insulating region $d\rho/dT < 0$ to the metallic region $d\rho/dT > 0$ is observed. The MR shows a maximum, and with increasing T, the position of the MR maximum in B increases for both GaAs-based (sample A) and GaN-based (sample B) 2DESs. Our data suggest that the EEI plays an important role in such a non-monotonic MR effect and in the temperature dependence of the resistivity.

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I. INTRODUCTION

The existences of a critical metal-insulator transition at zero magnetic field and of a non-monotonic magnetoresistivity (MR) [1–8] in two-dimensional electron systems (2DESs) have been extensively studied over the last few decades. Temperature-dependent corrections to the conductivity of 2DESs due to electron scattering by Friedel oscillations and the interaction constant F_0^σ have been considered to support such a non-monotonic metal-insulator transition and non-monotonic MR [9–14]. In 2DESs, the carriers are confined to move in a plane with a random potential. As the temperature is reduced to the diffusive region ($k_BT\tau/\hbar < 1$), a logarithmic increase of the resistivity (insulating behavior) is expected. According to the scaling theory of localization considering relatively weak Coulomb interactions, experimental results are in agreement with theoretical predictions that a weak electron-electron interaction (EEI) could increase the localization [15]. Moreover, in weakly-disordered systems $k_Fl \gg 1$, (where k_F and l are the Fermi momentum and the mean free path, respectively), experiments have shown that for strong coupling samples, with increasing electron density, the resistivity can cross from a region where $d\rho/dT < 0$ (insulating behavior) to a region where it decreases with decreasing temperature, *i.e.*, $d\rho/dT > 0$ (metallic behavior) [1–6,16–18].

The two mechanisms in the theory of quantum corrections to the Drude conductivity are the weak localization (WL) contribution (normally negative) and the EEI contribution. The contribution of the EEI depends on the value of the interaction constant F_0^σ [4,17]. The interaction constant F_0^σ can be understood as the ratio of the exchange to the kinetic energy, and it can be found from measurements of the magnetic susceptibility [19]. Theories predict that the non-monotonic resistivity $\rho(T)$ depends on a negative F_0^σ. The densities studied also play an important role in such a metal-insulator transi-

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tion. A window for a crossover exists when the electron density of a 2DES is varied. At relatively high densities, one can consider coherent electron scattering on the modulated density of electrons caused by an impurity, and one can attribute the metallic state to temperature-dependent impurity scattering caused by a modulation of the screening of the impurity potential [20,21].

The WL contribution comes from the interference of electron waves propagating in opposite directions along closed paths, and it depends on the temperature and the magnetic field. The WL correction is proportional to -ln(τ_ϕ). Here, τ_ϕ is the phase relaxation time, $\tau_\phi \propto T^{-P}$, with $p \sim 1$ in the dirty limit, and it is the timescale for a conduction electron to stay in a given, exact, one-electron energy eigenstate in the presence of static impurities. For 2DESs, small-energy-transfer electron-eletron (e-e) scattering is the dominant dephasing process, giving rise to $1/\tau_{\phi,ee} \propto T$ [22,23].

Recently, the non-monotonic MR of 2DESs in a perpendicular magnetic field B has been widely discussed [7, 8]. According to the classical Drude model, the longitudinal resistivity is independent of the perpendicular magnetic field [24]. However, a B-dependent MR property has been observed in many experiments [25–30]. Two mechanisms, WL and EEI, for quantum corrections to the Drude conductivity have been well established and, thus, can be used to explain the B-dependent MR in the diffusive region. The theory of EEI with short-range potential fluctuations has been widely discussed. The EEI correction in the diffusive region is proportional to $\ln(\hbar/k_BT\tau)$ and grows in amplitude as the temperature is decreased [4,31]. Zala *et al.* developed a theory for the temperature dependence of the conductivity by considering Friedel oscillations [11,12] to enhance the backscattering probability. Sedrakyan and Raikh (SR) showed that double scattering by Friedel oscillations in the ballistic region gave rise to a positive MR in a weak magnetic field, which could explain the non-monotonic MR [13].

II. EXPERIMENTS AND DISCUSSION

We used two different types of 2DESs, GaAs-based (sample A) and GaN-based (sample B) 2DESs, to study the non-monotonic, temperature-dependent resistivity and the non-monotonic magnetoresistivity. Sample A is an $Al_{0.33}Ga_{0.67}As/GaAs$ heterostructure grown by using molecular beam epitaxy (MBE). A standard Hall bar is mesa-etched with a channel width of 80 μm on this sample. The carrier concentration n and the electron mobility μ are 2.67 $\times$ 10^{11} cm^{-2} and 82300 cm^2/Vs, respectively, at 4 K. Sample B is grown by using metal-organic chemical-vapor deposition (MOCVD) on a sapphire substrate with the following layer sequence: a buffer layer, a 2.8-μm undoped GaN layer, a 67-nm Si-doped GaN layer, a 4.5-nm undoped GaN layer, a 3.5-nm undoped AlGaN layer, a 21-nm Si-doped AlGaN layer, a 3.5-nm undoped AlGaN layer and a 3 nm GaN cap layer. The longitudinal and the Hall MR were measured on Hall bars with length-to-width ratios of 5. The typical value of the current flow was 1 μA. At T = 4 K, the electron density was 1.23 $\times$ 10^{13} cm^{-2}, and the mobility was 3128 cm^2/V s.

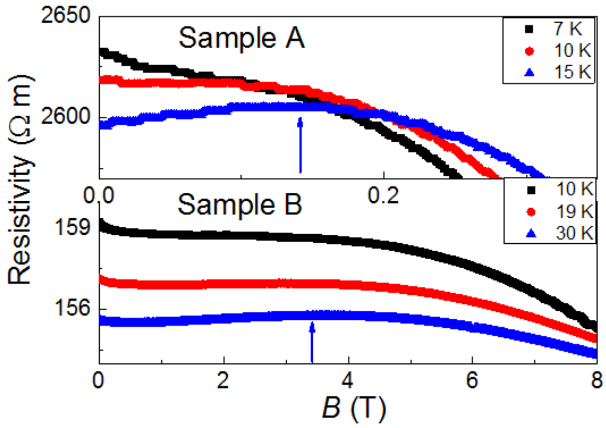

Fig. 1. (Color online) Temperature-dependent magnetoresistivity for both GaAs-based (sample A) and GaN-based (sample B) 2DESs.

In a magnetic field, the classical Drude conductivity tensor has the following form:

$$\sigma_D = \frac{ne\mu}{1+\mu^2B^2}\begin{pmatrix} 1 & \mu B \\ -\mu B & 1 \end{pmatrix} , \tag{1}$$

where n and μ are the electron density and mobility, respectively. A general formula for the longitudinal magneto-conductivity with correction terms is given by the expression

$$\sigma_{xx} = \sigma_0 + \delta\sigma_{xx}^{ee} + \delta\sigma^{WL} . \tag{2}$$

Here, $\sigma_0 = ne\mu$ is the Drude conductivity at a zero magnetic field; $\delta\sigma_{xx}^{ee}$ and $\delta\sigma^{WL}$ are the EEI correction term and the WL correction term, respectively. One specific feature of the EEI in the diffusive limit is the following:

$$\delta\sigma_{xx}^{ee} = -\frac{e^2}{2\pi^2\hbar}\left[1+3\left(1-\frac{\ln(1+F_0^\sigma)}{F_0^\sigma}\right)\right]\ln\left(\frac{\hbar}{k_BT\tau}\right) . \tag{3}$$

Figure 1 shows that both samples A and B exhibit a non-monotonic variation of the magnetoresistivity with increasing magnetic field. Moreover, a transition of the temperature-dependent resistivity from an insulator phase ($d\rho/dT < 0$) to a metallic phase ($d\rho/dT > 0$) is observed in the ballistic region $k_BT\tau/\hbar > 1$, as shown in Fig. 2. Because $\sigma_0 \gg \delta\sigma_{xx}^{ee}$, the normalized magnetoresistivity can be derived by inverting the conductivity matrix in a weak magnetic field ($\mu B \ll 1$) and has the form

$$\Delta\rho_{xx}(B,T)/\rho_0^2 \approx (1-\mu^2B^2)\delta\sigma_{xx}^{ee} . \tag{4}$$

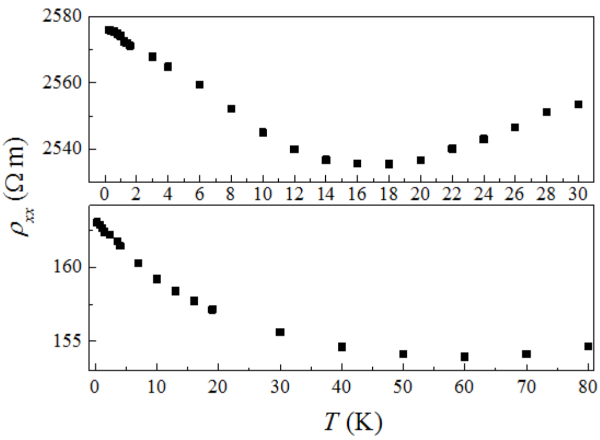

Fig. 2. Transition of the resistivity $\rho(T)$ from the insulator region $d\rho/dT < 0$ to the metallic region $d\rho/dT > 0$ when the temperature is increased. Top panel: sample A. Bottom panel: sample B.

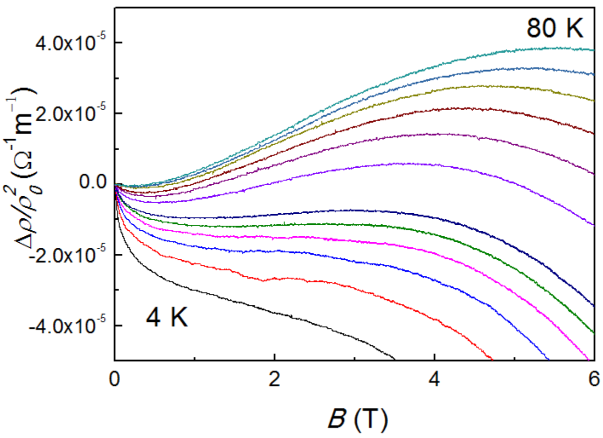

Fig. 3. (Color online) Renormalized resistivity $[\rho(B,T) - \rho(0,T)]/\rho^2(0,T)$ versus magnetic field for sample B.

From Eq. (4), the negative magnetoresistivity varies logarithmically with the temperature in the diffusive limit. For sample B ($k_F l \sim 160 \gg 1$), the normalized MR curves are shown in Fig. 3. For weak magnetic fields, $\mu B_{tr} < 1/k_F l$, one can see that applying a magnetic field perpendicular to the plane of a 2DES suppresses coherent back-scattering and that the suppression of the WL contribution gives rise to a negative magnetoresistivity.

The magnitude of the magneto-conductivity correction due to weak localization in two dimensions was quantified by Hikami *et al.* [32]. The magneto-conductivity in the absence of spin relaxation under the assumption that electron transport is diffusive is given by

$$\Delta\sigma_{WL} = \sigma_{xx}(B) - \sigma_{xx}(B=0) = -\frac{e^2}{2\pi^2\hbar}\left[\Psi\left(\frac{1}{2}+\frac{B_\phi}{B}\right) - \Psi\left(\frac{1}{2}+\frac{B_0}{B}\right)\right] , \quad (5)$$

where Ψ is the digamma function. B_0 and B_ϕ are characteristic magnetic fields related to the transport scattering rate and to the phase relaxation rate, respectively. An analysis of the WL curves has been found, in turn, to provide quantitative information on the electron dephasing mechanisms ($B_\phi = \hbar/4\pi D\tau_\phi$) [32–34]. At low temperatures, when the dephasing effect is dominated by electron-electron scattering, the phase coherence rate $1/\tau_\phi$ in a two-dimensional system is given by [33,34]

$$\frac{1}{\tau_\phi} = \frac{k_B T}{2\pi N_0 D\hbar^2}\ln(\pi D N_0 \hbar) , \quad T < \frac{\hbar}{k_B\tau} , \quad (6)$$

$$\frac{1}{\tau_\phi} = \frac{\pi k_B^2 T^2}{2\hbar E_F}\ln\left(\frac{E_F}{k_B T}\right) , \quad T > \frac{\hbar}{k_B\tau} , \quad (7)$$

where N_0 is the two-dimensional density of states and D is the diffusion constant.

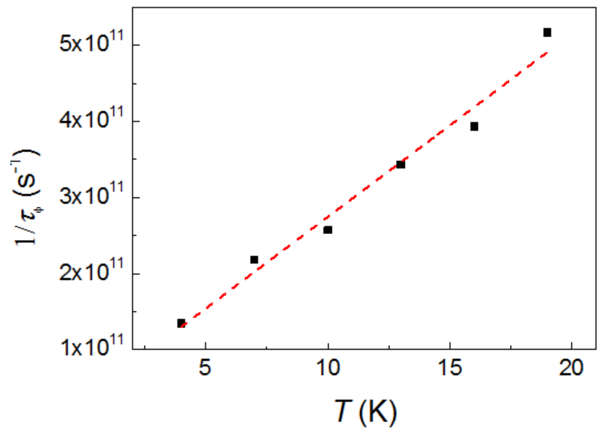

Fig. 4. (Color online) Dephasing rate is proportional to temperature. That is, the dephasing effect is dominated by electron-electron scattering.

Figure 4 shows that the dephasing rate is proportional to the temperature; that is, the dephasing effect is dominated by electron-electron scattering. As shown in Fig. 3, $\delta\sigma_{xx}^{ee}$ changes sign from positive to negative as the temperature increases when $B_{tr} < B < 1/\mu$.

In the Sedrakyan Raikh theory [13], the MR is found to arise from scattering of electrons on Friedel oscillations of the electron density around impurities. Double scattering from the field-modified Friedel oscillations was demonstrated to give a magnetoresistivity in the ballistic region ($\delta\sigma_{xx}^{ee} < 0$) [7]. Friedel oscillations are limited by the length $r_T = \nu_F/2\pi T$ rather than by the mean free path l. We plot the length r_T and the mean free path l in Fig. 5.

When Friedel oscillations become important for $rT \ll l$, a positive MR is found. We show the temperature dependence of the magnetoresistivity in fixed magnetic fields in Fig. 6(a). The value of $\rho(T)$ at a fixed magnetic field is larger than that at zero magnetic field $\rho(B=0,T)$, and the negative MR disappears when the temperature is higher than 30 K. The consequence of such for the EEI is related to the value of the interaction constant F_0^σ, which depends on the interaction strength. Zala *et al.* [11] showed that the correction

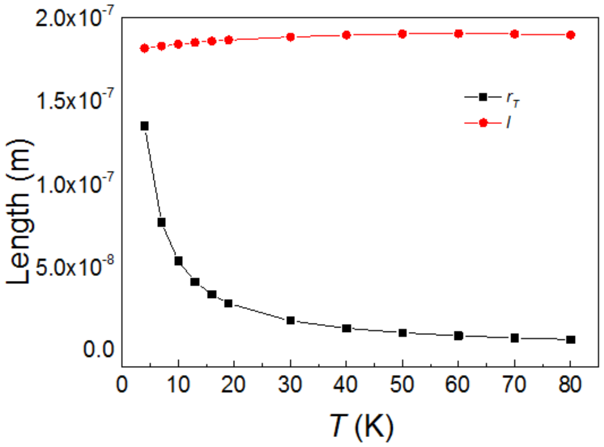

Fig. 5. (Color online) Mean free path and limit length of Friedel oscillations r_T versus T. At $T = 19$ K, $k_BT\tau/\hbar \sim 1$ for sample B.

was almost always monotonic, except for a narrow region $-0.45 < F_0^\sigma < -0.25$. Unlike the correction in the diffusive limit, the correction to the conductivity could change depending on the value of F_0^σ. This is due to competition between the universal Fock correction and the coupling specific Hartree contribution. For a stronger interaction, the Hartree correction should be modified to include higher-order processes [11,12]. In the diffusive region, using Eq. (3), we have obtained the slope that allows us to calculate the Hartree factor $F_0^\sigma = -0.105$ for temperatures lower than 4 K [28]. Note that an extra factor for the correction to conductivity, $\delta\sigma_{xx}^{ee}$, has been included in some papers [9, 11] with $k_Fl \gg 1$ systems. The corresponding expression can be expressed as

$$\begin{aligned}\delta\sigma_{xx}^{ee} &= -\frac{e^2}{2\pi^2\hbar}\left[1+3\left(1-\frac{\ln(1+F_0^\sigma)}{F_0^\sigma}\right)\right] \\ &\quad\times\left[\ln\left(\frac{\hbar}{k_BT\tau}\right)+\ln\left(\frac{k_Fl}{2}\right)\right] \\ &= -\frac{e^2}{2\pi^2\hbar}\left[1+3\left(1-\frac{\ln(1+F_0^\sigma)}{F_0^\sigma}\right)\right]\left[\ln\left(\frac{E_F}{k_BT}\right)\right]\end{aligned} \tag{8}$$

The reason for such a discrepancy is that the mean-free-path term and the Friedel oscillations can modify the transport mobility and, therefore, contribute not only to the σ_{xx} component but also to the σ_{xy} component [9,35].

$\delta\sigma_{xx}^{ee}$ vs $\ln(k_BT/E_F)$ is plotted in Fig. 6(b), and the slope gives the Hartree interaction constant. Our data show that such a linear relation could probably be extended to the ballistic region for $k_Fl \gg 1$ systems. Zala *et al.* [11] show that in the ballistic region, the temperature dependence of the conductivity is still governed by the same physical processes as the Altshuler-Aronov corrections - electron scattering by Friedel oscillations.

Both WL (quantum interference) and electron scattering by Friedel oscillations (backscattering) increase the resistivity [36–42]. With decreasing temperature, the metallic state is broken due to strong backscattering. For our samples, this strong backscattering due to Friedel oscillations happened in the ballistic region. However, as the temperature was decreased, the restrictive length for Friedel oscillations increased; thus, the backscattering probability due to Friedel oscillations decreased, but the quantum interference increased. Moreover, as the magnetic field was increased, the quantum interference decreased, but the backscattering probability (by Friedel oscillations) increased due to the curving scattering length caused by the magnetic field.

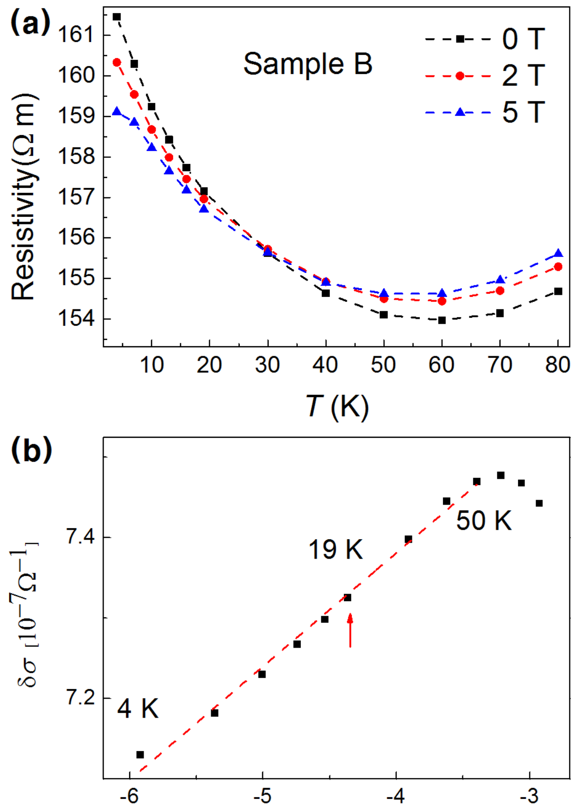

Fig. 6. (Color online) (a) Temperature dependence of the magnetoresistivity in fixed magnetic fields. The red dashed line is for a magnetic field of 2 T, which is chosen in the region at B_{tr} (~0.02 T) $< B < 1/\mu(\sim 3T)$ and the blue dashed line is for a magnetic field of 5 T, which is near to the magnetic field of the local maximum of MR. (b) $\delta\sigma$ vs. $\ln(k_BT\tau/E_F)$ plot. The slope gives the Hartree interaction constant.

III. CONCLUSION

We have reported a novel non-monotonic magnetoresistivity in both GaAs-based (sample A) and GaN-based (sample B) 2DESs in the ballistic region. A plot of the dephasing rate versus temperature shows electron-electron scattering to be dominant. A positive MR appears when the restrictive length of Friedel oscillations becomes smaller than the mean free path, as the theories predict. Our results show a transition of resistivity $\rho(T)$ from the insulator region $d\rho/dT < 0$ to the metallic region $d\rho/dT > 0$ in the ballistic region. The observation of a critical metal-insulator transition and a non-monotonic magnetoresistivity in 2DESs should be considered along with electron scattering by using Friedel oscillations and the interaction constant F_0^σ. Both the density (Friedel oscillations) and the interaction constant in systems with long mean free paths play important roles in such non-monotonic quantum transport.

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