Chinese Journal of Physics 82 (2023) 149–154

Contents lists available at ScienceDirect

Chinese Journal of Physics

journal homepage: www.sciencedirect.com/journal/chinese-journal-of-physics

Quantum Hall plateau-plateau transition revisited

Ching-Chen Yeh [a,1], Siang-Chi Wang [a,1], Shun-Tsung Lo [b,*], Gil-Ho Kim [c], D.A. Ritchie [d], Gottfried Strasser [e], Chi-Te Liang [a,f,g,*]

[a] Department of Physics, National Taiwan University, Taipei, 106, Taiwan
[b] Department of Electrophysics and Center for Emergent Functional Matter Science, National Yang Ming Chiao Tung University, Hsinchu, 300, Taiwan
[c] School of Electronic and Electrical Engineering, Sungkyunkwan Advanced Institute of Nanotechnology (SAINT) and School of Advanced Materials Science and Engineering, Sungkyunkwan University, Suwon, 440-746, Korea
[d] Cavendish Laboratory, J. J. Thomson Avenue, Cambridge, CB3 OHE, United Kingdom
[e] Institute of Solid State Electronics, TU Wien, Gusshausstrasse 25-25a, Vienna, 1040, Austria
[f] Center for Quantum Science and Engineering, National Taiwan University, Taipei, 106, Taiwan
[g] Taiwan Consortium of Emergent Crystalline Materials (TCECM), Taipei, 106, Taiwan

A R T I C L E I N F O

Keywords:
Quantum Hall plateau
Critical exponent
Transition
Two-dimensional critical magnetic field

A B S T R A C T

We observe a temperature-independent point B_c in the longitudinal resistance R_{xx} of a GaAs-based two-dimensional electron gas in the quantum Hall plateau-plateau transition. This allows us to revisit the spin-degenerate plateau-plateau transition and use the R_{xx} peak movement with respect to B_c at various temperatures T so as to calculate the critical exponent κ. We suggest that one measures the maximum of the derivative of Hall resistivity $d\rho_{xy}/dB \sim T^{-\kappa}$ as well as the movement of R_{xx} at different temperatures, allowing one to measure κs using two independent methods. When the peak movement is so large that the resistance peak is no longer in the vicinity of B_c, a largely deviated $\kappa = 0.54 \pm 0.04$ from the value (0.21 ± 0.02) in some spin-degenerate system can be measured.

1. Introduction

The plateau-plateau transition in the quantum Hall (QH) regime, which is perhaps one of the most studied quantum phase transitions, can be observed by changing the magnetic field B applied perpendicularly to the plane of a two-dimensional electron gas (2DEG) near absolute zero temperature T=0 [1,2]. Between two adjacent QH plateaux, there are a peak in the measured longitudinal resistance R_{xx} and a riser in the Hall resistivity ρ_{xy} [1]. In the pioneering work done by Wei and co-workers [1], by studying the maximum of $d\rho_{xy}/dB \sim T^{-\kappa}$ at various temperatures, the critical exponent κ was determined to be 0.42 ± 0.04 for a spin-split plateau-plateau transition [1,2]. We would like to point out that the exponent κ depends on spin/valley degeneracy. For a spin-degenerate plateau-plateau transition, κ is measured to be 0.21 ± 0.02 [3]. Moreover, we note that in the work of Wei *et al.,* no clear temperature-independent point in the longitudinal resistivity ρ_{xx} was observed [1]. Furthermore, the measured critical exponents cover a wide range of values ($0.15 \leq \kappa \leq 0.81$) [1,4-8], and it has been argued that the value of κ may not be universal [4]. On the other hand, it has been shown that in $Al_xGa_{1-x}As/Al_{0.33}Ga_{0.67}As$ heterostructures over a range of Al concentration x, for x between 0.65 %

* Corresponding authors.
E-mail addresses: stlo@nycu.edu.tw (S.-T. Lo), ctliang@phys.ntu.edu.tw (C.-T. Liang).
[1] These authors contributed equally to this work

https://doi.org/10.1016/j.cjph.2022.12.016
Received 10 September 2022; Received in revised form 19 December 2022; Accepted 31 December 2022
Available online 2 January 2023
0577-9073/

and 1.6 %, where the dominant contribution to disorder is from the short-range alloy potential fluctuations, perfect power-law scaling in the temperature range with a critical exponent $\kappa = 0.42 \pm 0.01$ is observed for a spin-split plateau-plateau transition [5,6]. In addition to experimental works and considerable efforts of numerical simulations over the years, a consensus on quantum Hall transition scaling behavior has not been reached [9–11].

It is worth mentioning that between a plateau-plateau transition, the peak position in B of the measured ρ_{xx} can *increase* with increasing T [1,12,13]. Such an effect was often overlooked but has been ascribed to a feature of scaling behavior of standard quantum Hall theory [12,13]. In order to further study such an interesting phenomenon, we measure a weakly disordered 2DEG. Moreover, we re-analyze some experimental data taken on a different 2DEG [14]. We find that it is possible to obtain the critical exponent κ from the movement of the resistance peak position in magnetic field with respect to a critical field B_c between two quantum Hall plateau at different temperatures [15]. However, in order to obtain the correct critical exponent, one must compare κ extracted from the peak movement in magnetic field with respect to B_c with that determined the maximum of $d\rho_{xy}/dB$ at different temperatures. If the peak movement goes way beyond the vicinity of the critical magnetic field, the measured κ may show large deviation from that determine from the usual method based on the maximum of $d\rho_{xy}/dB$ at different temperatures.

2. Sample preparation and measurement setup

Sample A was grown by molecular beam epitaxy (MBE) and consists of a 20-nm-wide $Al_{0.33}Ga_{0.67}As/GaAs/Al_{0.33}Ga_{0.67}As$ quantum well. The following layer sequence was grown on a GaAs (100) semi-insulating (SI) substrate: 50-nm-thick undoped $Al_{0.33}Ga_{0.67}As$, 20-nm-thick GaAs, 40-nm-thick undoped $Al_{0.33}Ga_{0.67}As$, 40- nm-thick doped $Al_{0.33}Ga_{0.67}As$, and finally a 17-nm-thick GaAs cap layer. Importantly, the growth of the 20-nm-wide GaAs quantum well was interrupted at its center; the wafer was cooled from 580 °C to 525 °C. The shutter over the In cell was opened for 80 s which allows the growth of 2.15 monolayer of InAs. A 5-nm-thick GaAs cap layer was then grown at 530 °C, before the substrate temperature was increased to 580 °C for the remainder of the growth [16,17]. In sample A, self-assembled InAs quantum dots, typically 4 nm in height and 28 nm in diameter, are formed near the center of the GaAs quantum well [16]. The charge density and mobility were $n = 1.30 \times 10^{15}$ m^{-2} and $\mu = 1.01$ m^2/Vs. For sample B, the MBE layer sequence on a GaAs (100) SI substrate was as follows: 30-nm-thick GaAs, 30 periods of a 2-nm AlAs/2-nm GaAs superlattice, 1-μm-thick GaAs, 20-nm-thick $Al_{0.33}Ga_{0.67}As$, A Si-doping layer with a concentration of 10^{18} cm^{-3}, 40-nm-thick $Al_{0.33}Ga_{0.67}As$, and finally a 5-nm-thick GaAs cap layer [14]. The charge density and mobility were $n = 3.24 \times 10^{15}$ m^{-2} and $\mu = 22.0$ m^2/Vs. Moreover, the contact resistances were measured to be around 25 Ω. The two devices were made with a Hall pattern by standard wet-etching processes and optical lithography. AuGeNi alloy was evaporated and annealed to form Ohmic contacts to the 2DEG. Low-temperature experiments were performed in a top-loading He^3 cryostat (Sample A) and a He^4 cryostat (Sample B). Standard ac magnetoresistance measurements were performed using phase-sensitive lock-in techniques.

3. Results and discussion

Fig. 1 shows the longitudinal resistivity and Hall resistivity measurements taken on Sample A at different temperatures. A clear ν=4 to ν =2 QH plateau-plateau transition is observed. Here ν is the Landau level filling factor. It is worth mentioning that we have observed the key feature, which is a critical magnetic field B_c in ρ_{xx} in the seminal work by Pruisken [15]. Such a critical point is not always observed in a 2D charge system, thereby hindering one from testing Pruisken's theory [1,13]. At different temperatures, the ρ_{xx} peak movement with respect to B_c in magnetic field (see Fig. 1) can be described by the following Eq. [15]

$$\ln(B^{max} - B_c) = \kappa \ln T + \text{const}, \tag{1}$$

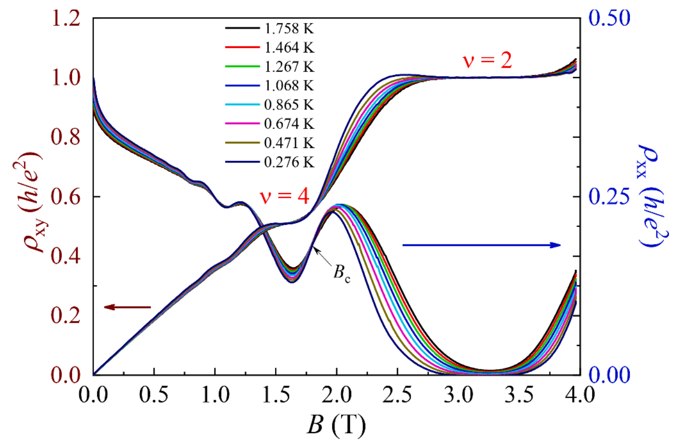

Fig. 1. Longitudinal and Hall resistivities of sample A at different temperatures. A critical magnetic field B_c in which ρ_{xx} is temperature-independent between the ν=4 and ν=2 quantum Hall states is observed.

where B_{max} corresponds to the peak position in magnetic field at different temperatures. By plotting $\ln(B^{max} - B_c)$ versus ln$T$, one can determine κ which is the slope of the linear fit. As shown in Fig. 2 (a), The standard method regarding the maximum of $d\rho_{xy}/dB \sim T^{-\kappa}$ at various temperatures allows us to measure $\kappa = 0.22 \pm 0.02$, close to the expected value 0.21. In contrast, the method regarding the peak movement yields $\kappa = 0.31 \pm 0.02$, which is slightly higher than the expected value of 0.21 as shown in Fig. 2 (b). Nevertheless, our experimental results, *for the first time*, indicate that one can measure the critical exponent κ following the method described by Eq. (1). Whether the determined κ is reliable or not is a different matter.

In order to further probe the plateau-plateau transition by the movement of the resistance peak between two adjacent QH states, we re-analyze the experimental data that we obtained previously (Sample B) [14]. Fig. 3 shows the longitudinal and Hall resistances of Sample B as a function of magnetic field at different temperatures. As shown in Fig. 3, two clear critical fields B_{c1} and B_{c2} can be observed. These allow us to measure the critical exponents for both the ν=8 to ν=6 and ν=6 to ν=4 plateau-plateau transitions using the method described by Eq. (1). Importantly, we can also determine the critical exponents using the results on the maximum of $d\rho_{xy}/dB$ at different temperatures independently. Such results are shown in Figs. 4 (a) and 4 (b). For the ν=8 to ν=6 plateau-plateau transition, both methods yield κ close to the expected value (0.21). For the ν=6 to ν=4 plateau-plateau transition, the Hall resistivity data allow us to determine $\kappa = 0.25 \pm 0.02$, which is close to the expected value. In contrast, the method using Eq. 1 yields a value $\kappa = 0.54 \pm 0.04$, which deviates from the expected value of 0.21 a lot. This critical value is close to the measured value for a GaAs-based hole gas [18]. In their case, the spin-orbit coupling is important [18]. However, in our study, we note that for sample B, the measured κ's for the 8-6 transition using both methods are both close to the value of 0.21 measured in some spin-degenerate charge systems [3]. Only when the peak movement is not in the vicinity of the crossing point, the measured $\kappa = 0.54 \pm 0.04$ deviates from that (0.23 ± 0.03) determined from the conventional method. Moreover, spin-orbit coupling is not significant in our GaAs-based two-dimensional electron systems. Therefore, here we suggest that the measured $\kappa = 0.54 \pm 0.04$ is not due to the spin-related effect, but

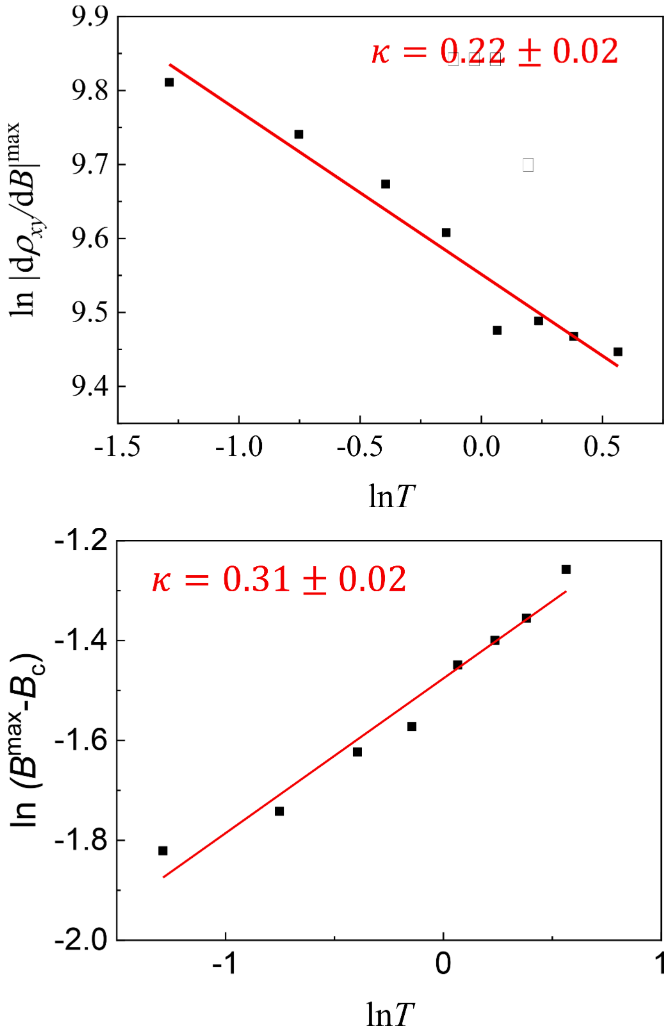

Fig. 2. (a) $\ln|d\rho_{xy}/dB|^{max}$ versus ln T for Sample A. The slope of the linear fit yields κ. (b) $\ln(B^{max} - B_c)$ versus ln T for Sample A. The slope of the linear fit allows us to calculate κ.

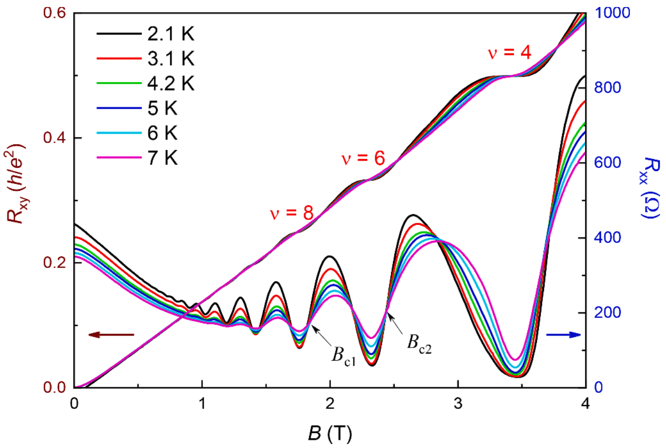

Fig. 3. Longitudinal resistance and Hall resistances of sample B at different temperatures. A critical magnetic field B_{c1} (B_{c2}) in which R_{xx} is temperature-independent between the ν=8 and ν=6 (ν=6 and ν=4) quantum Hall plateau-plateau transition is observed. Adapted from Ref. [14].

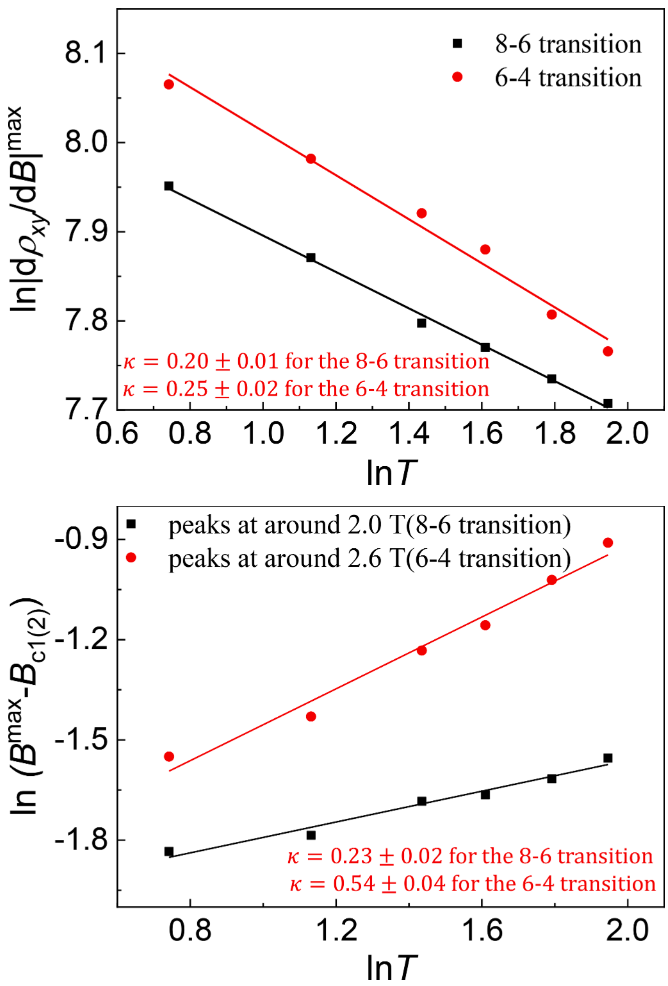

Fig. 4. (a) $\ln|d\rho_{xy}/dB|^{max}$versus ln$T$ for Sample B. The slopes of the linear fits yield κ for the 8-6 and 6-4 transitions. (b) $\ln(B^{max} - B_c)$ versus ln$T$ for Sample B. The slopes of the linear fits allow us to calculate κ for the 8-6 and 6-4 transitions.

possibly due to the fact that the peak movement is far away from the critical region. It has been suggested that in order to get a correct critical exponent, it is essential that the scaling analysis must be performed near the critical point [19]. Here we propose a possible reason for this. It has been suggested that in order to get a correct critical exponent, it is essential that the scaling analysis must be performed near the critical point [19]. In our case, the resistance peak movement is so large that the peak position is no longer in the vicinity of B_c, thereby resulting in a substantially larger κ than the expected value. Therefore, our experimental results indicate that in order to obtain the correct critical exponent κ, which is the key parameter in the QH plateau-plateau transition, one must measure both the longitudinal and Hall resistances at different temperatures so that one can determine κ using both the conventional method using the maximum of $d\rho_{xy}/dB$ (T) as well as the method based on the peak movement with respect to the critical magnetic field. In this work, the conventional method based on the Hall resistance data appears to be more reliable as κ is close to the value (0.21) in all cases.

4. Conclusions

In summary, we have demonstrated that by studying the ρ_{xx} peak movement with respect to the critical magnetic field, it is possible to extract the critical exponent κ in the quantum Hall plateau-plateau transition. In some case, the measured κ is close to the expected value of 0.21 for a spin-degenerate QH plateau-plateau transition. Nevertheless, the measured κ could be vastly different from that determined from the conventional method using the maximum of the derivative of Hall resistivity at different temperatures. Thus, our experimental data indicate the conventional method based on the Hall resistivity data at different temperatures is a more reliable method. We would like to suggest that further experimental work should be done on various 2D systems such as GaN-based 2DEGs [20–22], graphene-based 2D systems [23,24], and two-dimensional materials like InSe [25] in order to further study the plateau-plateau transition. Additionally, by comparing the values extracted from different methods, we might be able to propose a novel method to probe the intrinsic properties of a 2D system.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

We would like to thank Prof. Eugene Kogan for helpful discussions and for drawing our attention to Ref. [15]. We thank J. P. Bird, G. Bohra, J. C. Chen, E. Comfort, C. F. Huang, C. P. Huang, T.-Y. Huang, M.-G. Kang, L.-H. Lin, T. Y. Lin, and Y.-T. Wang for their experimental help at an early stage of this work. This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (No. 2019R1A2C2088719). C.-T.L. would like to thank the National Science and Technology Council (NSTC), Taiwan for financial support (grant numbers: NSTC 111-2119-M-002-007, NSTC 111-2622-8-002-001, NSTC 110-2112-M-002-029-MY3, and NSTC 108-2119-M-002-025-MY3).

References

[1] H.P. Wei, D.C. Tsui, M.A. Paalanen, A.M.M. Pruisken, Experiments on delocalization and university in the integral quantum Hall effect, Phys. Rev. Lett. 61 (1988) 1294–1296, https://doi.org/10.1103/PhysRevLett.61.1294.
[2] A.M.M. Pruisken, Universal singularities in the integral quantum Hall effect, Phys. Rev. Lett. 61 (1988) 1297–1300, https://doi.org/10.1103/PhysRevLett.61.1297.
[3] H.P. Wei, S.Y. Lin, D.C. Tsui, A.M.M. Pruisken, Scaling in spin-degenerate Landau levels in the integer quantum Hall effect, Phys. Rev. B 48 (1993) 11416–11419, https://doi.org/10.1103/PhysRevB.48.11416.
[4] S. Koch, R.J. Haug, K. von Klitzing, K. Ploog, Experiments on scaling in $Al_xGa_{1-x}As$/GaAs heterostructures under quantum Hall conditions, Phys. Rev. B 43 (1991) 6828–6831, https://doi.org/10.1103/PhysRevB.43.6828. R.
[5] Wanli Li, G.A. Csáthy, D.C. Tsui, L.N. Pfeiffer, K.W. West, Scaling and universality of integer quantum Hall plateau-to-plateau transitions, Phys. Rev. Lett. 94 (2005), 206807, https://doi.org/10.1103/PhysRevLett.94.206807. -1-206807-4.
[6] Wanli Li, C.L. Vicente, J.S. Xia, W. Pan, D.C. Tsui, L.N. Pfeiffer, K.W. West, Scaling in plateau-to-plateau transition: a direct connection of quantum Hall systems with the Anderson localization model, Phys. Rev. Lett. 102 (2009), 216801, https://doi.org/10.1103/PhysRevLett.102.216801. -1-216801-4.
[7] S. Koch, R.J. Haug, K. von Klitzing, K. Ploog, Size-dependent analysis of the metal-insulator transition in the integral quantum Hall effect, Phys. Rev. Lett. 67 (1991) 883–886, https://doi.org/10.1103/PhysRevLett.67.883.
[8] S. Koch, R.J. Haug, K. von Klitzing, K. Ploog, Experimental studies of the localization transition in the quantum Hall regime, Phys. Rev. B 46 (1992) 1596–1602, https://doi.org/10.1103/PhysRevB.46.1596.
[9] E.J. Dresselhaus, B. Sbierski, I.A. Gruzberg, Scaling collapse of longitudinal conductance near the integer quantum Hall transition, Phys. Rev. Lett. 129 (2022), 026801, https://doi.org/10.1103/PhysRevLett.129.026801. -1-026801-7.
[10] M.R. Zirnbauer, Marginal CFT perturbations at the integer quantum Hall transition, Ann. Phys. 431 (2021), 168559, https://doi.org/10.1016/j.aop.2021.168559. -1-168559-20.
[11] R. Bondesan, D. Wieczorek, M.R. Zirnbauer, Gaussian free fields at the integer quantum Hall plateau transition, Nucl. Phys. B 918 (2017) 52–90, https://doi.org/10.1016/j.nuclphysb.2017.02.011.
[12] C.F. Huang, Y.H. Chang, C.H. Lee, H.T. Chou, H.D. Yeh, C.-T. Liang, Y.F. Chen, H.H. Lin, H.H. Cheng, G.J. Hwang, Insulator-quantum Hall conductor transitions at low magnetic field, Phys. Rev. B 65 (2002), 045303, https://doi.org/10.1103/PhysRevB.65.045303. -1-045303-4.
[13] C.-W. Liu, C. Chuang, Y. Yang, R.E. Elmquist, Y.-J. Ho, H.-Y. Lee, C.-T. Liang, Temperature dependence of electron density and electron–electron interactions in monolayer epitaxial graphene grown on SiC, 2D Mater 4 (2017), 025007, https://doi.org/10.1088/2053-1583/aa55b9. -1-025007-7.
[14] S.-T. Lo, Y.-T. Wang, G. Bohra, E. Comfort, T.-Y. Lin, M.-G. Kang, G. Strasser, J.P. Bird, C.F. Huang, L.-H. Lin, J.C. Chen, C.-T. Liang, Insulator, semiclassical oscillations and quantum Hall liquids at low magnetic fields, J. Phys.: Condens. Matter 24 (2012), 405601, https://doi.org/10.1088/0953-8984/24/40/405601. -1-405601-7.

[15] A.M.M. Pruisken, Topological principles in the theory of Anderson localization, Int. J. Mod. Phys. B 24 (2010) 1895–1949, https://doi.org/10.1142/S0217979210064654.
[16] G.-H. Kim, C.-T. Liang, C.F. Huang, J.T. Nicholls, D.A. Ritchie, P.S. Kim, C.H. Oh, J.R. Juang, Y.H. Chang, From localization to Landau quantization in a two-dimensional GaAs electron system containing self-assembled In as quantum dots, Phys. Rev. B 69 (2004), 073311, https://doi.org/10.1103/PhysRevB.69.073311. -1-073311-4.
[17] T.-Y. Huang, C.-T. Liang, G.-H. Kim, C.F. Huang, C.-P. Huang, J.-Y. Lin, H.-S. Goan, D.A. Ritchie, From insulator to quantum Hall liquid at low magnetic fields, Phys. Rev. B 78 (2008), 113305, https://doi.org/10.1103/PhysRevB.78.113305. -1-113305-4.
[18] X. Wang, H. Liu, J. Zhu, P. Shan, P. Wang, H. Fu, L. Du, L.N. Pfeiffer, K.W. West, X.C. Xie, R.-R. Du, X. Lin, Scaling properties of the plateau transitions in the two-dimensional hole gas system, Phys. Rev. B 93 (2016), 075307, https://doi.org/10.1103/PhysRevB.93.075307. -1-075307-5.
[19] C.F. Huang, Y.H. Chang, H.H. Cheng, C.-T. Liang, G.J. Hwang, A study on the universality of the magnetic-field-induced phase transitions in the two-dimensional electron system in an AlGaAs/GaAs heterostructure, Physica E 22 (2004) 232–235, https://doi.org/10.1016/j.physe.2003.11.256.
[20] J.-H. Chen, J.-Y. Lin, J.-K. Tsai, H. Park, G.-H. Kim, J. Ahn, H.-I. Cho, E.-J. Lee, J.-H. Lee, C.-T. Liang, Y.F. Chen, Experimental evidence for Drude-Boltzmann-like transport in a two-dimensional electron gas in an AlGaN/GaN heterostructure, J. Korea. Phys. Soc. 48 (2006) 1539–1543. https://www.jkps.or.kr/journal/view.html?spage=1539&volume=48&number=6.
[21] D.R. Hang, C.-T. Liang, C.F. Huang, Y.H. Chang, Y.F. Chen, H.X. Jiang, J.Y. Lin, Effective mass of two-dimensional electron gas in an $Al_{0.2}Ga_{0.8}N$/GaN heterojunction, Appl. Phys. Lett. 79 (2001) 66–68, https://doi.org/10.1063/1.1380245.
[22] C.-T. Liang, S.-T. Lo, The direct insulator-quantum Hall transition, Chin, J. Phys. 52 (2014) 1175–1193, https://doi.org/10.6122/CJP.52.1175.
[23] K.S. Novoselov, A.K. Geim, S.V. Morozov, D. Jiang, M.I. Katsnelson, I.V. Grigorieva, S.V. Dubonos, A.A. Firsov, Two-dimensional gas of massless Dirac fermions in graphene, Nature 438 (2005) 197–200, https://doi.org/10.1038/nature04233.
[24] Y. Zhang, Y.-W. Tan, H.L. Stormer, P. Kim, Experimental observation of the quantum Hall effect and Berry's phase in graphene, Nature 438 (2005) 201–204, https://doi.org/10.1038/nature04235.
[25] Denis A. Bandurin, Anastasia V. Tyurnina, Geliang L. Yu, Artem Mishchenko, Viktor Zólyomi, Sergey V. Morozov, Roshan Krishna Kumar, Roman V. Gorbachev, Zakhar R. Kudrynskyi, Sergio Pezzini, Zakhar D. Kovalyuk, Uli Zeitler, Konstantin S. Novoselov, Amalia Patanè, Laurence Eaves, Irina V. Grigorieva, Vladimir I. Fal'ko, Andre K. Geim, Yang Cao, High electron mobility, quantum Hall effect and anomalous optical response in atomically thin InSe, Nat. Nanotechnol. 12 (2017) 223–227, https://doi.org/10.1038/nnano.2016.242.